1.4.3 Simplifying Boolean Expressions Questions

1.

i. Simplify the Boolean expression **¬(A Λ B)** using De Morgan's First Law.

[1]

ii. Simplify the Boolean expression **¬¬A** using double negation.

[1]

iii. Simplify the Boolean expression **X v (Y ∧ Z)** using distribution.

[2]

2. A Boolean expression for a logic system is shown below:
P = ¬ (¬ A v ¬ B)
Simplify this Boolean expression so that it does not include any negation. You must explain which Boolean algebra rule(s) you are using at each step.

1.4.3 Simplifying Boolean Expressions Questions

3. A user can be granted premium access to FezFlix, a movie streaming service only if they meet one of the following criteria:

- The user has an active monthly subscription **and** a verified email address.
- The user is on a free trial **and** is referred by an existing premium member.

Let:

P be Premium Access is granted
A be User has an active monthly subscription
B be User has a verified email address
C be User is on a free trial
D be User is referred by an existing premium member

The criteria for premium access to FezFlix can be written as:

$P \equiv (A \wedge B) \vee (C \wedge D)$

FezFlix has a **promotional code** that unlocks a **bonus movie stream** when the code is used in conjunction with either a **gift card purchase** or a **social media share**.

Let:

E be Promotional code is used
F be Gift card is purchased
G be FezFlix is shared on social media
R be Bonus movie stream is unlocked

This bonus movie stream logic could be written as:

$R \equiv (E \wedge F) \vee (E \wedge G)$

i. Complete the truth table below.

E	F	G	E∧F	E∧G	(E∧F)∨(E∧G)
1	1	1			
1	1	0			
1	0	1			
1	0	0			
0	1	1			
0	1	0			
0	0	1			
0	0	0			

[4]

ii. Simplify the expression $(E \wedge F) \vee (E \wedge G)$

[2]

1.4.3 Simplifying Boolean Expressions Questions

FezFlix offers new release films to users on a pre-order basis, allowing the film data to be downloaded to the user's device 5 days before the official streaming release date. FezFlix needs to implement a robust logic check to ensure that a film cannot be played before its official release date.

Using the rules for manipulating Boolean expressions, **simplify the following expression** which represents a complex access control check for a user attempting to stream a pre-downloaded film.

Let the variables represent components of the access control system:

- A be The official release date has passed
- B be User's device clock is synchronised with the FezFlix server
- C be Device has a valid decryption key

4. Simplify the following access control expression:

$A \wedge B \vee A \wedge (B \vee C) \vee B \wedge (B \vee C)$

[4]

END OF QUESTION PAPER

Mark scheme

<table>
<tr><th colspan="3">Question</th><th>Answer/Indicative content</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>1</td><td></td><td>i</td><td>¬A v ¬B</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>ii</td><td>A</td><td>1</td><td></td></tr>
<tr><td></td><td></td><td>iii</td><td>• 1 mark for (A v B)
• 1 mark for ∧ (A v C)

(A v B) ∧ (A v C)</td><td>2</td><td></td></tr>
<tr><td></td><td></td><td></td><td>Total</td><td>4</td><td></td></tr>
<tr><td>2</td><td></td><td>i</td><td>• Identify De Morgan’s AND/OR rule
• Identify double negation rule
• Correct final answer to give A ∧ B</td><td>3</td><td></td></tr>
<tr><td></td><td></td><td></td><td>Total</td><td>3</td><td></td></tr>
<tr><td>3</td><td></td><td>i</td><td>
<table>
<tr><th>E</th><th>F</th><th>G</th><th>E∧F</th><th>E∧G</th><th>(E∧F)∨(E∧G)</th></tr>
<tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
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1 mark for each of the pairs of rows.</td><td>4</td><td></td></tr>
<tr><td></td><td></td><td>ii</td><td>(F∨G) ∧ E

One mark for the (F∨G)

One mark for the ∧ E</td><td>2</td><td>Accept:

(G∨F) ∧ E

E ∧ (F∨G)

E ∧ (G∨F)</td></tr>
<tr><td></td><td></td><td></td><td>Total</td><td>6</td><td></td></tr>
<tr><td>4</td><td></td><td></td><td>Original Expression
A∧B∨A∧(B∨C)∨B∧(B∨C)</td><td></td><td>For 4 marks - 1 mark for each bullet completed correctly.</td></tr>
</table>

			- A∧B∨(A∧B∨A∧C)∨(B∧B∨B∧C) Distributive Law (X∧(Y∨Z)=X∧Y∨X∧Z) **3.**A∧B∨A∧B∨A∧C∨B∨B∧CIdempotence Law (B∧B=B) **4.**A∧B∨A∧C∨B∨B∧CIdempotence Law (X∨X=X) - combining A∧B∨A∧B **5.**B∨B∧CAbsorption Law (X∨(X∧Y)=X) - B absorbs B∧C **6.**B∨A∧B∨A∧CCommutative Law (reordering terms) **7.**B∨A∧CAbsorption Law (X∨(X∧Y)=X) - B absorbs A∧B		